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THE STUDY OF BASE HEATING BY RADIATION FROM EXHAUST GASES

by

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EXPERIMENTAL PROGRAM

Assembly of the high-temperature furnace and the optical system to be used for measurements of infrared absorption of gases was completed. Another test run with argon inside and helium outside of the zirconia tube was made to a temperature of 2800°F. No fracture occurred on the zirconia tube this time, probably due to the high thermal conductivity of helium outside. The furnace and auxiliary components functioned satisfactorily. A traverse of the platinum and platinum-rhodium thermocouple shows a drop of gas temperature of about 400°F between the temperature at the center of the tube and that near the water-cooled windows. Modification of the graphite heating element is planned for reducing this drop in temperature when more data on the temperature distributions are available.

A test run on the optical system has been made to measure the absorption spectrum of CO_2 at room temperatures. Except for a few minor changes which are being considered, the optical system performed satisfactorily. The measurements obtained agree very well with existing data in the literature.

In the immediate future spectral data will be obtained for CO_2 at temperatures in the 2000 to 3000°F range. These results will be used to evaluate the effect of non-isothermal conditions in the test section and to guide modification of the heating element.

ANALYTICAL PROGRAM

The investigation of the calculation of radiation from a gas body non-uniform in temperature is being continued. A procedure for evaluating the radiation incident on the center of the base plane from a non-isothermal non-gray cylindrical gas body is included in this progress report. The results show that the effect of non-uniform temperature on the radiation incident on the base plane could be quite significant. The magnitude of this effect, however, depends strongly on the actual absorption spectrum of the gas under consideration.

The study of radiative energy transfer from a cylindrical gas body to the inner and outer base regions is of interest in connection with the combustion in rocket engines[1] and radiative base heating from rocket exhaust plumes.[2] All these studies, however, are based on the physical model of a gray isothermal gas. For an isothermal gas body, the assumption of gray gas presents no serious problem in actual calculations as the non-gray emissivity can be obtained by numerically integrating over the wavelengths of the gray-gas emissivity with the absorption coefficient given by the actual infrared absorption spectrum. The situation becomes quite different in the case of non-isothermal gas bodies, since the contribution due to each absorption band will change as a result of the temperature dependence of Planck's distribution. The present work is to analyze and show the effect of non-isothermal and non-gray gas on the emissivity at the center of the base of a cylindrical gas body.

ANALYSIS

The physical system under consideration is a right circular cylindrical gas body, as shown in Fig. 1. The gas is of uniform composition and non-scattering. The spectral absorption coefficient will be assumed to be independent of temperature. The spectral radiative energy flux per unit area and time incident on the center of the base is given by:[1]

$$q_\lambda = 2\, a_\lambda \int_0^h \int_0^{\pi/2} e_{b\lambda}\, e^{-a_\lambda \ell} \sin\beta \, d\beta \, dz \qquad (1)$$

in terms of dimensionless quantities defined by

$$\epsilon_\lambda = q_\lambda/\sigma T_o^4 \qquad E_{b\lambda} = e_{b\lambda}/\sigma T_o^4 \qquad A_\lambda = a_\lambda r_o$$
$$R = r/r_o \qquad Z = z/r_o \qquad H = h/r_o \qquad L = \ell/r_o \tag{2}$$

Eq. (1) becomes

$$\epsilon_\lambda = 2 A_\lambda \int_0^H \int_0^{\pi/2} E_{b\lambda}\, e^{-A_\lambda L} \sin\beta \, d\beta \, dZ \tag{3}$$

where ϵ_λ is called the apparent emissivity. The spectral emissive power $e_{b\lambda}$ (and $E_{b\lambda}$) is a function of position in the gas body because of the non-uniform temperature field. In the following analysis, the temperature will be assumed to be a function of axial distance only. Sinilar analysis can easily be made for the case of non-uniform radial-temperature distribution. Noting that $L = Z/\cos\beta$ and integrating with respect to β in Eq. (3), there follows

$$\epsilon_\lambda = 2 A_\lambda \int_0^H E_{b\lambda} \left[E_2(A_\lambda Z) - (Z/\sqrt{1+Z^2})\, E_2(A_\lambda \sqrt{1+Z^2})\right] dZ \tag{4}$$

where E_2 is the exponential integral defined by

$$E_n(x) = \int_1^\infty e^{-xt}\, t^{-n}\, dt \tag{5}$$

and the emissive power $E_{b\lambda}$ in explicit expression is

$$E_{b\lambda} = \frac{C_1}{\sigma T_o^4 \lambda^5 [\exp(C_2/\lambda T) - 1]} \tag{6}$$

It is obvious that the integral in Eq. (4) is so involved that only numerical evaluation is possible. In particular, the complexity due to the non-uniform temperature $T(Z)$ in the expression of $E_{b\lambda}$ is formidable. However, in order to understand the functional dependence of ϵ_λ

on the non-uniform temperature distribution and the spectral absorption coefficient, an analytical expression of ϵ_λ is required. To achieve the above purpose, several approximations have to be employed. First, the Planck distribution will be approximated by the Wien distribution, i.e.,

$$E_{b\lambda} \approx (C_1/\sigma T_o{}^4)\, \lambda^{-5} \exp\,[-C_2/\lambda T] \tag{7}$$

The above equation gives excellent approximation for Eq. (6) in the range $\lambda T \leq 0.3$ cm °K and smaller values in the range $\lambda T > 0.3$ cm °K.[1] Secondly, the exponential integral will be approximated by a simple exponential function, i.e.,

$$E_2(x) \approx (3/4) \exp\,(-3x/2) \tag{8}$$

where the areas and the first moments of both sides are equal, respectively. This approximation has been used in numerous studies of radiative-transfer problems.[3]

For the purpose of illustrating the effect of non-uniform temperature distribution, a simple expression is chosen for the temperature,

$$(T/T_o) = 1/(1+bZ) \tag{9}$$

where b is a dimensionless parameter characterizing the temperature distribution. The particular expression makes the exponential in Eq. (7) much more tractable mathematically. Substitution of Eq. (9) into Eq. (7) yields

$$E_{b\lambda} = E_{b\lambda}(o) \exp\,(-B_\lambda Z) \tag{10}$$

where $B_\lambda = (C_2 b/\lambda T_o)$ is a parameter indicating the effect of non-uniform temperature distribution on the spectral emissive power.

With E_2 and $E_{b\lambda}$ given in Eqs. (8) and (10), respectively, Eq. (4) can be rearranged

$$\epsilon_\lambda = \left(\frac{3A_\lambda \; E_{b\lambda}(o)}{2B_\lambda + 3A_\lambda} \right) \left\{ 1 - \exp\left[- \left(B_\lambda + \frac{3A_\lambda}{2} \right) H \right] - \int_1^{\sqrt{1+H^2}} \exp\left[- B_\lambda \sqrt{\xi^2 - 1} - \frac{3A_\lambda}{2} \xi \right] d\xi \right\} \tag{11}$$

the integral in Eq. (11) can be approximated without large error by

$$\int_1^{\sqrt{1+H^2}} \exp\left[- B_\lambda \sqrt{\xi^2 - 1} - \frac{3A_\lambda}{2} \xi \right] d\xi \approx \int_1^{\sqrt{1+H^2}} \exp\left[- \left(B_\lambda - \frac{3A_\lambda}{2} \right) \xi \right] d\xi \tag{12}$$

since the areas under the two exponential curves are quite close for $\xi > 1$. Therefore, Eq. (11) is given by

$$\epsilon_\lambda = \left(\frac{3A_\lambda \; E_{b\lambda}(o)}{2B_\lambda + 3A_\lambda} \right) \left\{ 1 - \exp\left[- \left(B_\lambda + \frac{3A_\lambda}{2} \right) \right] - \exp\left[- \left(B_\lambda + \frac{3A_\lambda}{2} \right) H \right] + + \exp\left[- \left(B_\lambda + \frac{3A_\lambda}{2} \right) \sqrt{1+H^2} \right] \right\} \tag{13}$$

For the special case of a semi-infinite cylindrical gas body $(H \to \infty)$, Eq. (13) becomes

$$\epsilon_\lambda = \left(\frac{3A_\lambda \; E_{b\lambda}(o)}{2B_\lambda + 3A_\lambda} \right) \left\{ 1 - \exp\left[- \left(B_\lambda + \frac{3A_\lambda}{2} \right) \right] \right\} \tag{14}$$

If, on the other hand, the temperature is uniform, there follows from Eq. (13)

$$\epsilon_\lambda = E_{b\lambda} \left[1 - \exp\left(- \frac{3A_\lambda}{2} \right) - \exp\left(- \frac{3A_\lambda H}{2} \right) + \exp\left(- \frac{3A_\lambda}{2} \sqrt{1+H^2} \right) \right] \tag{15}$$

which would be the classical result of Schmidt[4] when similar exponential-integral approximations were employed. The approximate results based on Eq. (15) are found in good agreement with the exact numerical results given by Schmidt.

The total emissivity is given by

$$\varepsilon = \int_0^\infty \epsilon_\lambda \, d\lambda \tag{16}$$

and must be evaluated numerically with A_λ given by actual absorption spectrum. Under the gray-gas assumption, $A_\lambda = A$ independent of wavelengths (but noting that B_λ is still a function of λ), the integral in Eq. (16) with ϵ_λ given by Eq. (13) can be easily integrated. The expression, however, is very lengthy and is not given here.

RESULTS

To demonstrate the effect of non-gray and non-isothermal gas, calculations were made for the emissivity at the center of the base of a semi-infinite cylindrical gas body $(H \to \infty)$. Similar characteristics are expected for the case of finite cylinders. Shown in Fig. 2 is the apparent emissivity as a function of b, the parameter characterizing non-uniform temperature distribution. The solid line represents the gray-gas result, while the dotted line represents that of the actual non-gray gas of CO_2. The absorption data of CO_2 were taken from Ref. 5. The results show that for a gray gas the effect of non-uniform temperature is quite significant. That the effect is smaller in the case of CO_2 is due to the increasing contribution of 15-micron band as the temperature decreases. This indicates that the actual absorption spectrum plays an important role in the determination of emissivity of a non-isothermal gas body.

REFERENCES

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NOMENCLATURE

a_λ	spectral absorption coefficient
A_λ	dimensionless spectral absorption coefficient $(A_\lambda = a_\lambda r_o)$
b	characteristic parameter of temperature defined by Eq. (9)
B_λ	$(C_2 b/\lambda T_o)$
C_1, C_2	First and Second constants in Planck distribution
E_n	exponential integral defined by Eq. (5)
$e_{b\lambda}$	spectral black-body emissive power
$E_{b\lambda}$	dimensionless spectral black-body emissive power $(E_{b\lambda} = e_{b\lambda}/\sigma T_o^4)$
h	height of the cylindrical gas
H	dimensionless height $(H = h/r_o)$
ℓ	distance from origin
L	dimensionless distance from origin $(L = \ell/r_o)$
q_λ	spectral radiative heat flux
r	radial distance $(r = \ell \sin \beta)$
r_o	radius of the cylindrical body
R	dimensionless radial distance $(R = r/r_o)$
T	absolute temperature
T_o	temperature at the base $(Z = 0)$
z	axial distance
Z	dimensionless axial distance $(Z = z/r_o)$
β	polar angle
Σ_λ	spectral apparent emissivity $(\epsilon_\lambda = q_\lambda/\sigma T_o^4)$
ε_λ	apparent emissivity defined by Eq. (16)
λ	wave length
σ	Stefan-Boltzmann constant

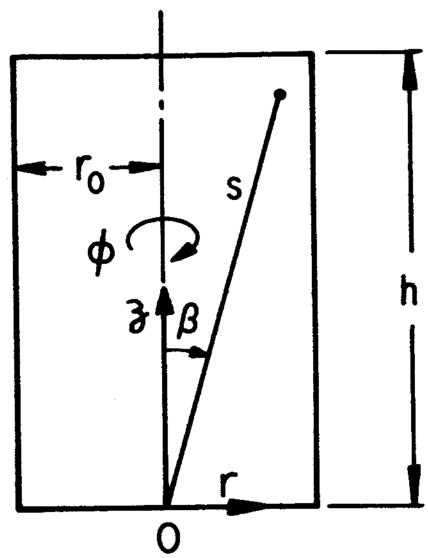

FIG. I RIGHT CIRCULAR CYLINDRICAL GAS BODY

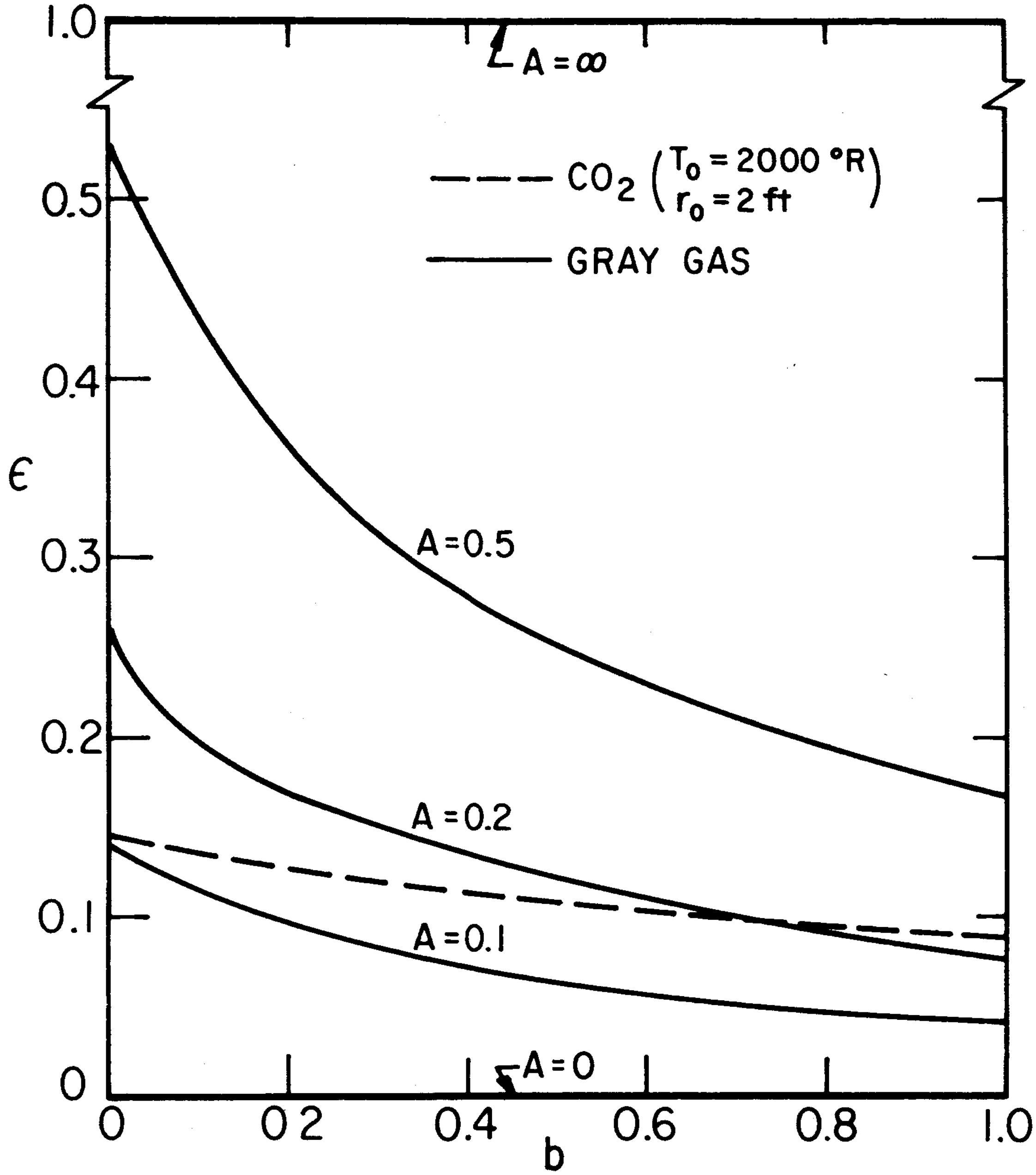

FIG. 2 APPARENT EMISSIVITY AT THE CENTER OF THE BASE OF A NON-ISOTHERMAL SEMI-INFINITE CYLINDRICAL GAS